

VOL.14, NO. 10
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DEOEMBER 1933


RAISING PAVEMENT BY MUD JACKING

# PUBUC ROADS … Highway Research 

Issued by the

UNITED STATES DEPARTMENT OF AGRICULTURE BUREAU OF PUBLIC ROADS<br>G. P. St. CLAIR, Editor

Volume 14, No. 10
December 1933
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# Laboratory Tests Assist in the Selection of Materials Suitable for Use in Mud Jack Operations 

Reported by A. M. WINTERMYER, Assistant Highway Engineer, Division of Tests, United States Bureau of Public Roads

IN THE routine procedure for testing subgrade soils which has been described previously, ${ }^{1}$ it was desired to include tests sufficient in number and varied enough in character to identify all of the various soils apt to be encountered in highway construction in all parts of the country and place each soil in the proper subgrade group as indicated by its physical characteristics.
Much of the soil testing is now done for one of the following purposes: (a) Determining the suitability of
tial features of an investigation of the suitability of soils for special purposes.

## Materials for mud Jacking must have certain

 CHARACTERISTICSA satisfactory mixture of soil, portland cement, and water for use in raising settled areas of road surfaces by mud jacking (see fig. 1) must be of such a character that it can be readily forced through the mud jack. It must possess qualities which will enable it to spread freely


Figure 1.-Low Place in Concrete Pavement Being Ratsed to Grade by Mud Jacking.
soil or soil material for some particular use such as binder in sand-clay and gravel roads, filler in bituminous road surfacing or soil for mud jack operations; (b) distinguishing between the good and the undesirable varieties of individual groups of surfacing materials such as limerock, caliche, shale, etc.; (c) identifying the properties of materials such as rock powders, diatomaceous earths, bentonites, etc., which, as admixtures may assist in the stabilization of both subgrades and soil road surfaces.

In testing materials for a special use the tests may be limited to those required to disclose the particular characteristics essential for the particular use. Studies to determine testing procedure consist of three distinct steps as follows:
(a) Analysis to determine what is to be required of the material for the use intended.
(b) Determination of the dominating characteristics satisfying these requirements.
(c) Selection of the particular tests best suited to furnish the desired information on these dominating characteristics.

The following suggested procedure in the study of soils for use in mud jack operations illustrates the essen-

[^0]over the subgrade as the separation between the road surface and subgrade increases during the pumping operation, and it must prevent appreciable settlement of the raised area of pavement after the pumping operation.

The mixture must be sufficiently plastic to permit its being forced readily through the pump and the hose connecting the pump with the opening in the pavement. Such a state of fluidity may be obtained with a variety of mixtures ranging from the smoothest cohesive pastes to those which have an appreciable degree of harshness. All of the mixtures which are fluid enough to perform satisfactorily in the pump, however, may not be capable of spreading to the desired extent over the subgrade. This is because the conditions controlling the spread of the mixture over the subgrade differ somewhat from those controlling the flow of the mixture through the pump and hose.

In passing through the pump and hose the mixture is confined to a definite channel of comparatively large size and only the proper combination of pump pressure and fluidity of mixture are required. When the paste reaches the subgrade, however, its flow is not restricted to any particular cross section. It is free to form in layers whose ratios of thickness to area of distribution depend to a very considerable extent on the frictional
resistance of the subgrade and the under face of the road slab which bound the shallow openings penetrated by the paste.

The smaller this ratio, the more uniform is the slab support both during and subsequent to the pumping operation. As the ratio increases there is an increasing tendency of mixtures to accumulate under certain slab areas, especially at the location where the paste is forced through the pavement.

The frictional resistance depends not only on the characteristics of the subgrade and the under side of the road slab but also upon the harshness of the paste.

The more cohesive mixtures spread most readily in a thin layer over a large area. With increasing harshness of mixtures there is an increasing tendency for the paste to form in accumulations leaving unsupported areas which are productive of cracking.

Harshness of mixture affects the frictional resistance of pump and hose but within the range of mixtures having the desired fluidity, the difference in harshness has but little practical significance, insofar as the pumping operation is concerned. Mixtures not sufficiently fluid are objectionable primarily because of their tendency to jam in the pump but they may also contribute to unsatisfactory distribution under the slab. Harsh mixtures are objectionable primarily because of their tendency to accumulate under limited slab areas and it is possible for them to be harsh enough to jam in the pump. Mixtures having harshness sufficient to jam in the jack are easily recognized and should be discarded. Experience indicates that a mixture considered by an experienced operator to be sufficiently fluid for use in a pump will spread satisfactorily over the subgrade.

Raised areas of pavement may settle after the pumping ceases as a result of several causes. The weight of the pavement may cause the viscous paste to flow out as soon as the pump stops or the paste may shrink on loss of moisture. The paste or soil mixture must be stable enough to support the pavement and loads produced by traffic immediately after pumping ceases and must resist shrinking upon loss of moisture.

The essential qualifications of a mixture for use in mud jacking can be stated as follows:

1. The paste should be fluid enough to be readily pumped.
2. It should be free enough from harshness to spread to the desired extent over the subgrade.
3. The paste, even in the high state of fluidity required for satisfactory performance in the pump, must not be subject to detrimental shrinkage upon loss of moisture.
4. The paste, after being properly placed, must change rapidly from a highly fluid, smooth-flowing state to one which is stable enough to resist lateral flow under considerable pressure.

> CHARACTERISTICS OF SUITABLE MATERIALS STATED

The fluid paste-forming properties of a mixture are furnished by the finer soil particles which comprise the silt and the clay fractions. Sand increases the stability of the mixture and reduces the shrinkage upon drying. The harshness of the mix is furnished by the sand fraction. Generally, the larger the sand particles or the higher the sand content, the harsher will be the soil paste.

A sufficient amount of sand to furnish all of the stability required at the conclusion of the pumping would
introduce harshness so great that the mixture could not be forced through the pump. It is not possible to use enough sand to eliminate all detrimental shrinkage.

A rapid increase in stability and elimination of detrimental shrinkage must be obtained with an admixture capable of permanently stabilizing the mixture to the desired extent. Portland cement, when added in sufficient amount, serves well for this purpose.

Studies of topsoil roads indicate that the presence of coarse sand particles (those retained on the no. 60 sieve) contributes to the hardness of a soil surface and increases its resistance to abrasion. When abrasive action is absent, fine sand particles (those passing the no. 60 sieve) may be just as efficient as coarse sand in stabilizing the soil structure. The harshness caused by fine sand is materially less than that caused by coarse sand. Therefore, fine sand seems better than the coarse sand for use in mud jack operations.

A suitable soil must be of such character that it will slake readily in water to form a mixture of uniform consistency. It must be free from glue-like colloids which do not soften readily when mixed with water and are productive of objectionable packing of the mixture in the mud pump.

Soil for use in mud jacking should consist of fine sand, silt, and clay in such proportions that the paste formed from the silt and clay will completely encompass all of the sand particles so as to eliminate detrimental harshness of the mixture. The silt and clay fraction should be free of gluey colloids productive of high-water capacity, high plasticity, and excessive shrinkage and, in addition, should be of the inactive character required to form a paste of the desired consistency and smoothness with minimum additions of water.

Generally, the greater the shrinkage properties of the soil, the larger will be the cement admixture required to prevent excessive shrinking. Admixture of cement sufficient to accomplish this purpose is considered sufficient to prevent detrimental flow of the paste after placing.

TESTS SHOULD BE SELECTED WHICH WILL DISCLOSE DESIRED CHARACTERISTICS
The tests selected for studying the properties of the mixtures under discussion should furnish information on the following:
(a) The grading of the materials.
(b) The shrinkage properties of the mixture.
(c) The character of the fine fraction of the soil.
(d) The fluidity of the mixture.
(e) The harshness of the mixture.

The mechanical analysis, the shrinkage limit, the liquid limit, and the preparation of flow curves of soils, are suggested as complying with these requirements.

Complete information on mechanical analysis, the shrinkage limit, the liquid limit, and flow curves has been given in previous reports. ${ }^{2}$. Therefore, the tests are referred to but briefly in this report. The essential features of the mechanical analysis are well understood and require no discussion. Results of mechanical analyses are shown graphically in figure 2.

The shrinkage limit is defined as the moisture content at which further evaporation does not produce further decrease in the volume of the soil. As water evaporates from a soil mixture, capillary tension acting like a taut skin on the surface of the soil mass
${ }^{2}$ Reports on subgrade soil studies, reprinted from Public Roads vol 12, nos. 4 , 5, 7, and 8, and Research on the Atterberg Limit of Soils, Public Roads, vol. 13, no. 8,


Figure 2.-Results of Mechanical Analysis of Soll Samples Showing Cumulative Percentages by Weight.
gradually draws the soil particles closer together until the shrinkage limit is reached. At this moisture content the resistance of the soil to further compression just equals the force exerted by the evaporating moisture. Below the shrinkage limit the resistance to compression exceeds the compressing force. Therefore, further evaporation below the shrinkage limit is not productive of further decrease in the volume of the soil mass. Soil mixtures used in mud jacking are not apt to shrink appreciably when their shrinkage limits are equal to or exceed the moisture content required to give the mixture the desired fluidity.
The liquid limit is the moisture content at which a groove (see fig. 3) separating two parts of a divided soil cake is just closed by a given number of shocks. This test is indicative of a particular degree of stability or consistency of the soil. In the routine or hand method of test, water or soil powder as required is added to the soil sample until the desired consistency, that requiring 10 shocks to close the soil groove, is obtained.

In the mechanical method (see fig. 4), the number of shocks required to close the groove with the soil at several consistencies are determined and plotted against the corresponding moisture contents to form "flow" curves. The liquid limit as determined by the hand method is the same as the moisture content corresponding to 25 shocks in the mechanical method. The number of shocks required to close the soil groove is a measure of the stability of the mixture and the flow curves, which are straight lines on semilog plots, disclose the relation between the moisture content and the degree of fluidity or flowability of the mixtures (see fig. 5).

It follows from this that the number of shocks or fraction of a shock corresponding to a particular moisture content on the flow curve extended becomes a quantitative measure of the consistency of the mixture at this moisture content. Therefore, all soils, at the moisture content corresponding to a given number of blows (including fractions of a blow) on the flow curves, should have the same degree of fluidity. Consequently, the number of blows indicative of the fluidity of mixtures required for mud-jack purposes as determined by tests on a comparatively few soils may serve as an index with which the required water content of soils generally may be determined.

The ratio of the liquid limit to the clay content discloses the relative activity of the clay fraction. Liquid limits approximately equal to the clay content indicate the presence of inactive fine particles capable of forming pastes with additions of relatively small amounts of water. The more the liquid limit exceeds the clay


DIVIDED SOIL CAKE BEFORE TEST


SOIL CAKE AFTER TEST
Figure 3.-Upper-Divided Soil Cake Before Receiving 10 Light Blows at Location Indicated by Arrow. LowerLateral Flow (Shear Failure) of Cake During Test.


Figure 4.-Mechanical Device for Making the Liquid Limit Test.
content the more the presence of undesirable gluey colloids is indicated.
The ratio of the shrinkage limit to the liquid limit indicates the shrinkage properties of the soil. The smaller the shrinkage limit as compared with the liquid limit the greater will be the tendency for the soil to shrink and consequently the greater will be the amount of cement required to prevent the shrinkage from becoming objectionable.

## LABORATORY PROCEDURE ILLUSTRATED

Table 1 gives the results of laboratory tests of four soils which have been used in mud-jack operations with various results. Sample no. S-6361 is a Virginia soil used in mud jacking on the Mount Vernon Memorial Highway. This soil was selected by an experienced operator because of its ability to form a paste of the desired consistency and smoothness. The other three

Table 1.-Results of laboratory tests on soils used in mud jacking

| Sample no. | Mechanical analysis, percentage by weight |  |  |  |  | Liquid limit | $\begin{gathered} \text { Shrink- } \\ \text { age } \\ \text { limit } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} \text { Passing } \\ \text { no. } 40 \\ \text { sieve } \end{gathered}$ | $\begin{gathered} \text { Coarse } \\ \text { sand- } \\ 2.25 \mathrm{~mm} \end{gathered}$ | $\left\lvert\, \begin{gathered} \text { Fine } \\ \text { sand- } \\ 0.25 \mathrm{to} \\ 0.05 \mathrm{~mm} \end{gathered}\right.$ | $\left\lvert\, \begin{gathered} \text { Silt- } \\ 0.05 \text { to } \\ 0.005 \mathrm{~mm} \end{gathered}\right.$ | Clay- smaller than 0.005 mm |  |  |
| S-6361 <br> S-7100 <br> S-7462 <br> S. 7463 | $\begin{aligned} & 99 \\ & 98 \\ & 99 \\ & 99 \end{aligned}$ | 5 <br> 9 <br> 3 <br> 5 | $\begin{aligned} & 27 \\ & 27 \\ & 10 \\ & 63 \end{aligned}$ | $\begin{aligned} & 43 \\ & 35 \\ & 29 \\ & 24 \end{aligned}$ | $\begin{array}{r} 25 \\ 29 \\ 58 \\ 8 \end{array}$ | 26 33 72 16 | $\begin{array}{r} 21 \\ 15 \\ \text { (1) } 9 \end{array}$ |

Pat did not shrink in test.


Figure 5.-Flow Curves of Soils Tested for Suitability for Mud Jacking.
samples represent Texas soils, the performance of which is reported to have been as follows:

That represented by sample no. S-7100 formed a mixture which worked well in the pump. Sample no. S-7462 has a high clay content and this soil could not be formed into a smooth paste and jammed in the pump. Sample no. S-7463 has a high sand content and formed a mixture so harsh that it "packed" both in the pump and under the pavement.
The absence of coarse sand in an appreciable amount is the only conspicuous feature common to all four of the soils. Samples no. S-6361 and no. S-7100 have somewhat similar gradings and their flow curves (fig. 4) are approximately parallel. Sample no S-7463 was so harsh that the mechanical device for liquid limit determination could not be used. A liquid limit of 16 was determined by the hand method. The presence of gluey colloids in sample no. S-7462 made it difficult to obtain uniform mixtures of water with this soil even in the laboratory which made it necessary to determine the liquid limit of this soil by the hand method.

The soil represented by sample no. S-6361 seems particularly well suited for mud jacking and its essential characteristics may serve as a guide in studies of soils for this purpose. This soil is representative of well graded and thoroughly weathered topsoils with little or no material retained on the no. 40 sieve. It is stable in road surfaces in both wet and dry weather but erodes badly in cut or fill faces. The fine sand ( 27 percent) furnishes stability without producing harshness of mix. The clay ( 25 percent), is relatively inactive. This is disclosed by the low liquid limit of 26 as compared with
a typical liquid limit of about 36 for average soils containing 25 percent of clay. The property which causes this low liquid limit is responsible for the erosion referred to above and also for relatively low shrinkage properties. The grading of this soil is uniform and the mechanical analysis, when plotted as shown in figure 2, approximates a straight line. (The equation of this line is $P=38(\log d+3)$ in which $P$ equals the percentage of particles smaller than the particle size $d$.)

Its liquid limit is practically equal to the clay content and its shrinkage limit is about 81 percent of the liquid limit. This latter value is significant because, on the average, shrinkage limits equal to or greater than about 75 percent of the liquid limits are indicative of inactive soils in which shrinkage is not important. The shrinkage limit is somewhat smaller than the liquid limit, indicating the presence of cohesive paste-forming particles. At present it is not known whether the slope of the flow curves, figure 5 , has any special significance.

The moisture content required to furnish the necessary fluidity of this soil, as determined by trial in the mud-jack operations, was found to be about 40 percent without cement admixture and about 42 percent with the cement admixture. Figure 5 shows that these moisture contents furnish a consistency corresponding to 0.1 blow on the flow curve extended.

The effect of cement admixtures in raising the shrinkage limit is shown in table 2. In these determinations, the soil and cement were mixed with 42 percent of water, molded into pats, and then allowed to dry to constant weight. No shrinkage of the pat occurred when the addition of cement became equal to 7 percent of the weight of the soil. Therefore it is safe to assume that the mixture in service will not shrink when containing cement in this amount.

Table 2.-Effect of cement admixtures upon the shrinkage limit of the soil represented by sample no. S-6361

| Cement <br> admix- <br> ture, per- <br> cent by <br> weight | Shrinkage <br> limit |
| :---: | :---: |
|  |  |
| 1 | 29 |
| 3 | 31 |
| 5 | 38 |
| 7 | $(1)$ |

${ }^{1}$ Pat did not shrink in test.
Upon this basis, the suggested mixture would contain about 100 pounds of dry soil, 7 pounds of portland cement, and 45 pounds or about $5 \frac{1}{2}$ gallons of water.
Sample no. S-7100 with a shrinkage limit of 15 and a required moisture content of about 48 percent (value corresponding to 0.1 in fig. 4) will shrink more than soil no. S-6361 with a shrinkage limit of 21 and a moisture content of mixture of about 42 percent. More cement would have to be added to S-7100 than to S-6361 to entirely eliminate the possibility of shrinkage. Since soils S-7463 and S-7462 could not be formed into pastes sufficiently fluid to be tested in the liquid limit device it is quite evident that they are not suitable for mud jack operations. In addition the low shrinkage limit of 9 for soil S-7462 indicates the presence of objectionable colloids in dominating amounts. The absence of shrinkage in the test of soil S-7463 suggests the absence of the cohesive paste-forming fine soil particles.

# Analytical Tools for Judging Results of Structural Tests of Concrete Pavements 

By H. M. WESTERGAARD, Professor of Theoretical and Applied Mechanics, University of Illinois, Urbana, Ill.

INFORMATION concerning stresses in pavements is needed for the purpose of establishing policies of design, for the purpose of imposing proper limits on the loads that may be transported, and for the purpose of apportioning the cost of pavements to the different kinds of traffic, with a view toward rational taxation of vehicles of various weights. The last reason has been a particular incentive in conducting structural studies of pavements during recent years.

Knowledge of the structural behavior of concrete pavements depends on tests and theory. Results of a theory were reported in a paper in 1926. ${ }^{1}$ This theory, in the main, was supported by the experimental data available at the time. It supplies answers to the question, what deflections and stresses may be expected when the subgrade is within a range of normal conditions, without unusual heaving and irregularity. This theory was used by the Bureau of Public Roads in planning a series of tests of pavement slabs. The tests were carried out by the Bureau of Public Roads at Arlington, Va., in 1932. The theory, having served in planning the tests, will naturally also furnish tools for examining the results of the tests. The tests suggest certain adaptations of the theory. There will be needed some restatements of analytical results, and some supplementing and modification of the theory.

## MEASURES OF STIFFNESS IN THE ORIGINAL THEORY

In the formulas used to demonstrate the original theory, as presented in 1926, various quantities appear as measures of stiffness, which are assumed to be constant in each particular case. The modulus of elasticity, $E$, and Poisson's ratio, $\mu$, of the concrete were introduced as constants. Furthermore, the resistance of the subgrade was expressed by a quantity, $k$, which is called the modulus of subgrade reaction. This modulus is defined as the reaction per unit of area per unit of deflection, and it may be measured in pounds per square inch per inch; or, pounds per cubic inch. It was admitted that this modulus could not be expected to be an actual constant of the material, expressing properties of the subgrade independently of the slab. But it was found that a rather wide variation of $k$ led to relatively small variations of the stresses, and consequently the modulus $k$ was introduced as a quantity which might be assumed to be constant for a given combination of slab, subgrade, and position of the load.

From $E, \mu, k$, and the thickness of the slab, $h$, a further quantity was obtained, a distance $l$, which is called the radius of relative stiffness. This distance appears in a number of the formulas and is defined by the equation,

$$
\begin{equation*}
l^{4}=\frac{E h^{3}}{12\left(1-\mu^{2}\right) k} \tag{1}
\end{equation*}
$$

[^1] Westergaard, Public Rnads, vol. 7, no. 2, April 1926.

## FORMULAS FOR THE STRESSES

In the previous paper approximate formulas for stresses in the interior of the area and at the edge (equations 11 and 12) were stated for the special case in which $E=3,000,000$ pounds per square inch and $\mu=0.15$. To make the theory usable in interpreting results of new tests, it is desirable to restate these formulas, so that they may be used for any values of $E$ and $\mu$. A third approximate formula, for the stress at a corner (equation 7), was stated for any values of $E$ and $\mu$.

The following notation is used:
$P=$ load.
$a=$ (1) radius of a small circle over the area of which $P$ is assumed to be distributed uniformly when the load is at a considerable distance from the edges; (2) radius of a small semicircle over the area of which $P$ is assumed to be distributed uniformly when the load is at the edge, the center of the semicircle being at the edge; (3) distance of the point of application of the resultant of the load from each of the two edges at a rectangular corner when $P$ is close to this corner (the distance from the corner being $a_{1}=a \sqrt{2}$ ).
$\sigma_{i}=$ maximum tensile stress at the bottom of the slab directly under the load $P$, when $P$ is at a considerable distance from the edges.
$\sigma_{e}=$ maximum tensile stress at the bottom of the slab at the edge due to a load at the edge.
$\sigma_{c}=$ maximum tensile stress at the top of the slab in a diagonal direction due to a load at a corner. $b=$ radius depending on $a$ and $h$.
The radius $b$ may be computed by the following approximate formulas (equation 8 in the previous paper):

$$
\begin{array}{r}
b=\sqrt{1.6 a^{2}+h^{2}}-0.675 h \text { when } a<1.724 h \ldots \ldots(2) \\
b=a \text { when } a>1.724 h \ldots \ldots \tag{3}
\end{array}
$$

The set of formulas for stresses becomes:
Stress in the interior of the area, at the bottom:

$$
\begin{equation*}
\sigma_{i}=0.275(1+\mu) \frac{P}{h^{2}} \log _{10}\left(\frac{E h^{3}}{k b^{4}}\right) \tag{4}
\end{equation*}
$$

Stress at the edge, at the bottom:

$$
\begin{equation*}
\sigma_{e}=0.529(1+0.54 \mu) \frac{P}{h^{2}}\left[\log _{10}\left(\frac{E h^{3}}{k b^{4}}\right)-0.71\right] \tag{5}
\end{equation*}
$$

Stress at the corner, at the top:

$$
\begin{equation*}
\sigma_{e}=\frac{3 P}{h^{2}}\left[1-\left(\frac{12\left(1-\mu^{2}\right) k}{E h^{3}}\right)^{0.15}(a \sqrt{2})^{0.6}\right]- \tag{6}
\end{equation*}
$$

With $E=3,000,000$ pounds per square inch and $\mu=0.15$ equations 4 and 5 give the same results as equations 11 and 12 in the previous paper. Equation 6 is a transcription of equation 7 in the previous paper

## NEW COEFFICIENTS PROPOSED

It is proposed to introduce a new coefficient, $K$, defined by the relation

$$
\begin{equation*}
K=k l_{-} \tag{7}
\end{equation*}
$$

The coefficient $K$, like $k$, will be a measure of the resistance of the subgrade. The reason for the proposal is the expectation that $K$ will be less dependent on the stiffness of the slab than is $k$. If the depressions of circular bearing blocks under a moderate load are inversely proportional to the diameter, then the product of the corresponding $k$ and the diameter would be constant. Correspondingly, in case of the slab, it is at least a possibility that the product of the most plausible value of $k$ times the linear dimension $l$ will be fairly constant for a given condition of the subgrade. It appears desirable, therefore, to supplement the formulas in terms of $k$ by a set of equivalent formulas in terms of $K$. The truth may lie between the two extreme cases of a constant $k$ and a constant $K$.

It is suggested that $K$ be called the coefficient of subgrade stiffness. It is noted that $K$ may be measured in pounds per square inch. For example, $k=50$ lb. in. ${ }^{-3}$ and $l=30 \mathrm{in}$. give $K=1,500 \mathrm{lb}$. in. ${ }^{-2}$

The following relations are derived from equations 1 and 7:

$$
\begin{align*}
& l^{3}=\frac{E h^{3}}{12\left(1-\mu^{2}\right) K^{-}}  \tag{8}\\
& K=\frac{E h^{3}}{12\left(1-\mu^{2}\right) l^{3}}-  \tag{9}\\
& \frac{K^{4}}{k^{3}}=\frac{E h^{3}}{12\left(1-\mu^{2}\right)} \tag{10}
\end{align*}
$$

It is proposed furthermore, to introduce a coefficient $D$, defined as

$$
\begin{equation*}
D=k l^{2} \ldots \ldots \ldots \tag{11}
\end{equation*}
$$

or

$$
\begin{equation*}
D=K l \tag{12}
\end{equation*}
$$

and to call this coefficient the deflection modulus of the pavement. The reason for this proposal is the consistent occurrence of the quantity $k l^{2}$ in the formulas for the deflections. In equations and diagrams in the previous paper the deflections are stated in the general form:

$$
\begin{equation*}
z=c \frac{P}{k l^{2}} \tag{13}
\end{equation*}
$$

where $c$ is a pure number in each case. For example, when $P$ is applied at the interior of the area of the slab, and the deflection $z$ is measured directly under the load, equation 14 in the previous paper gives $c=\frac{1}{8}$. In this case, if the pavement is considered as a spring, $8 k l^{2}$ is the modulus of the spring. With $D$ introduced, the deflections will be restated as

$$
\begin{equation*}
z=c \frac{P}{D} \tag{14}
\end{equation*}
$$

The following further relations involving $D$ are derived from the preceding equations:

$$
\begin{equation*}
D=\sqrt{\frac{E h^{3} k}{12\left(1-\mu^{2}\right)}}=\sqrt[3]{\frac{E h^{3} K^{2}}{12\left(1-\mu^{2}\right)}} \tag{15}
\end{equation*}
$$

$$
\begin{align*}
& k=\frac{12\left(1-\mu^{2}\right) \boldsymbol{\rho}^{2}}{E h^{3}}  \tag{16}\\
& K=\sqrt{\frac{12\left(1-\mu^{2}\right) D^{3}}{E h^{3}}}  \tag{17}\\
& E=\frac{12\left(1-\mu^{2}\right) D l^{2}}{h^{2}} \tag{18}
\end{align*}
$$

RESTATEMENT OF THE FORMULAS FOR THE STRESSES IN TERMS OF THE NEW COEFFICIENT OF SUBGRADE STIFFNESS

Since $1-\mu^{2}$ does not vary a great deal, this quantity may be computed as if $\mu$ were constant, equal to 0.15 , which gives $1-\mu^{2}=0.9775$. Then equations 4 and 5 may be restated as follows by use of equation 10 :

Stress in the interior of the area, at the bottom:
$\sigma_{i}=1.1(1+\mu) \frac{P}{h^{2}}\left[\log _{10}\left(\frac{h}{b}\right)+\frac{1}{3} \log _{10}\left(\frac{E}{K}\right)-0.089\right]-$
Stress at the edge, at the bottom:

$$
\begin{align*}
\sigma_{e}= & 2.117(1+0.54 \mu) \frac{P}{h^{2}}\left[\log _{10}\left(\frac{h}{b}\right)\right. \\
& \left.+\frac{1}{3} \log _{10}\left(\frac{E}{K}\right)-0.2666\right] \ldots \tag{20}
\end{align*}
$$

The stress at the corner, at the top becomes, by restatement of equation 6 ,

$$
\begin{equation*}
\sigma_{c}=\frac{3 P}{h^{2}}\left[1-\left(\frac{12\left(1-\mu^{2}\right) K}{E h^{3}}\right)^{0.2}(a \sqrt{2})^{0.5}\right] \tag{21}
\end{equation*}
$$

## DERIVATION OF THE CONSTANTS FROM TESTS

Six quantities assumed constant under given conditions occur in the equations which have been stated: $E, \mu, k, K, D$, and $l$. Since a variation of Poisson's ratio, $\mu$, has only a relatively small influence, it is sufficient to know an approximate value of $\mu$. There remain then the five quantities $E, k, K, D$, and $l$. If two of them can be determined to begin with by a direct examination of experimental data, then equations will be available by which the remaining three may be computed. Table 1 shows what equations are to be used.

Table 1.-Key to the computations of 3 of the constants $E, k, K, D$, and l from the remaining ?

| Known beforehand | Computation by equations |
| :---: | :---: |
| $E$ and $k=$ | $K$ by (10), $D$ by (15), $l$ by (1). |
| $E$ and $K$ | $k$ by (10), $D$ by (15), l by (8). |
| $E$ and $D$ | $k$ by (16), K by (17), $\ell$ by (18) or (11) or (12). |
| $E$ and $l$ | $k$ by (1), $K$ by (9), $D$ by (18) or (11) or (12). |
| $D$ and $l$ | $E$ by (18), $k$ by (11), $K$ by (12). |

The modulus of elasticity, E, may be obtained with a reasonable degree of certainty from tests of cylinders and control beams. The deflection modulus, $D$, may be obtained from observed maximum deflections by equation (14), with the values of $c$ as given in the previous paper. The radius of relative stiffness, $l$, appears as a unit of the horizontal scales in the diagrams of deflections in the previous paper, and may be obtained, therefore, by comparing the diagrams of deflections
observed at a series of points with the diagrams of deflections in the previous paper. By fitting both the horizontal and the rertical scales of a theoretical diagram of deflections of a series of points to the corresponding diagram of observed deflections, so as to make the two curves coincide as nearly as possible, $D$ and $l$ may be determined together.

When strains have been measured, and $E$ and $\mu$ are known approximately, the corresponding stresses may be computed from the strains. It would seem possible thereafter to compute $k$ or $K$ from equations 4 to 6 or 19 to 21. Inspection of these equations shows, however, that values of $k$ and $K$ determined in this way will be very sensitive to small variations of the stresses. It appears preferable, therefore, to assign rarious definite values to $k$ or $K$ and thereafter compare the experimental and analytical results.

In dealing with the stresses it is to be remembered that $\sigma_{i}$ and $\sigma_{e}$ are tensile stresses at the bottom of the slab. With small values of the radius, $a$, the compressive stresses at the top must be expected to show a somewhat greater concentration and therefore greater maximum values than would be found for the corresponding tensile stresses at the bottom.

SUPPLEMENTARY THEORY BASED ON REDISTRIBUTION OF THE SUBGRADE REACTIONS
This supplementary theory is limited to the case of a load applied in the interior of the area of the slab. In the original theory the reaction of the subgrade per unit of area at each point was assumed to be proportional to the deflection of the slab at that point. If the subgrade acts as a continuous body, it is to be expected that the reactions of the subgrade will be more closely concentrated around the load than are the deflections. Accordingly a set of corrections of the deflections will be introduced which may account for a redistribution of the reactions with an increase of the reactions near the load and a decrease of the reactions at greater distances from the load.

The following notation is used:
$r=$ radial distance measured horizontally from the point of application of the resultant of the load $l$ ?
$L=$ maximum value of $r$ within which adjustments are made.
$z=$ deflection according to the original theory.
$z^{\prime}=$ superimposed supplementary deflection.
$z^{\prime \prime}=z+z^{\prime}=$ resultant deflection.
$z^{\prime}, z_{i}^{\prime \prime}=$ values of $z^{\prime}$ and $z^{\prime \prime}$ at the point $r=0$.
$k: z=$ reaction per unit of area according to the original theory.
$q=$ supplementary reaction per unit of area, accounting for the deflection $z^{\prime}$.
$k z+q=$ resultant reaction per unit of area.
$\psi_{1}=$ value of $q$ at $r=0$.
$\epsilon_{i}^{\prime}=$ strain at the hottom of the slab at $r=0$ due to the deflection $z^{\prime}$.
$\sigma_{i}{ }_{i}=$ supplementary stress at the bottom of the slab at $r=0$, corresponding to the deflections $z^{\prime}$.
$\sigma^{\prime \prime}{ }_{i}=\sigma_{i}+\sigma_{i}^{\prime}=$ resultant stress at the bottom of the slab at $r=0, \sigma_{i}$ being defined by equation 4 or equation 19.
$Z=-\frac{z^{\prime} i}{z_{i}}=$ ratio of reduction of the maximum deflection.

The following formulas are proposed for the supplementary deflections:

$$
\begin{align*}
z^{\prime}=z^{\prime}\left(1-\frac{r^{2}}{L^{2}}\right)^{10} \text { when } r<L \ldots  \tag{22}\\
z^{\prime}=0 \text { when } r>L \tag{23}
\end{align*}
$$

Instead of the power 10 some other power might be used, but the power 10 appears to be suitable. A plausible value of $L$ may be $\overline{\delta L}$.

The deflections $z^{\prime}$ are caused by the reactions $q$, which may be interpreted as loads taken positive upward.

According to the theory of slabs ${ }^{2}$ the upward load per units of area, that is, the reaction, may be stated as

$$
\begin{equation*}
q=-\frac{E h^{3}}{12\left(1-\mu^{2}\right)} \Delta^{2} z^{\prime} \ldots \ldots \ldots \tag{24}
\end{equation*}
$$

in which $\Delta$ is the operator $\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}, x$ and $y$ being reetangular coordinates. In the present case it is preferable to use the equivalent expression for $\Delta$ in terms of the radius vector $r$, namely, ${ }^{3}$

$$
\begin{equation*}
\Delta=\frac{d^{2}}{d r^{2}}+\frac{1}{r} \frac{d}{d r} \tag{25}
\end{equation*}
$$

Equations 22, 24, and 25 give

$$
\begin{equation*}
q=q_{i}\left(1-18 \frac{r^{2}}{L_{i}^{2}}+45 \frac{r^{4}}{L^{\ddagger}}\right)\left(1-\frac{r^{2}}{L^{2}}\right)^{6}-\cdots \tag{26}
\end{equation*}
$$

in which $q_{i}$, the value directly under the load, is

$$
\begin{equation*}
q_{i}=-\frac{240 E h^{3} z^{\prime} i}{\left(1-\mu^{2}\right) L^{4}-} \tag{27}
\end{equation*}
$$

It is noted that the integral of $q$ over the whole area is zero. That is, the total vertical force added to $P$ is zero, and the reactions are merely redistributed.

The curvature of the slab in any direction, at $r=0$, due to the supplementary deflections $z^{\prime}$, becomes

$$
\begin{equation*}
\left[\frac{d^{2} z^{\prime}}{d r^{2}}\right]_{T=0}=\frac{20 z^{\prime}}{L^{2}}- \tag{28}
\end{equation*}
$$

The corresponding strain at the bottom of the slab is

$$
\begin{equation*}
\epsilon_{i}^{\prime}=\frac{10 h z^{\prime} i}{L^{2}} \tag{29}
\end{equation*}
$$

and the corresponding stress is

$$
\begin{equation*}
\sigma_{i}^{\prime}=\frac{10 E h_{i}^{\prime}}{(1-\mu) L_{i}} \tag{30}
\end{equation*}
$$

By use of equation 18 this stress is restated as

$$
\begin{equation*}
\sigma_{i}^{\prime}=\frac{120(1+\mu) D z^{\prime}}{h_{i}^{2}}\binom{1}{L}^{2} \tag{31}
\end{equation*}
$$

The deflection directly under the load according to the original theory is given by equation 14 in the previous paper, and is

$$
\begin{equation*}
z_{i}=\frac{P l^{2}}{8 I} \tag{32}
\end{equation*}
$$

[^2]It will be expedient to express the supplementary deflection at the same point in terms of a ratio of reduction $Z$, as

$$
\begin{equation*}
z_{i}^{\prime}=-Z_{z i}=-Z \frac{P}{8 I} \tag{33}
\end{equation*}
$$

Then the resultant deflection directly under the load becomes

$$
\begin{equation*}
z^{\prime \prime}{ }_{i}=z_{i}+z_{i}^{\prime}=(1-Z) \frac{P}{8} \bar{D} \tag{34}
\end{equation*}
$$

Equations 31 and 33 give

$$
\begin{equation*}
\sigma_{i}^{\prime}=-\frac{15(1+\mu)}{h^{2}} \ell P\left(\frac{l}{L}\right)^{2} \tag{35}
\end{equation*}
$$

The resultant maximum stress at the bottom of the slab is $\sigma^{\prime \prime}{ }_{i}=\sigma_{i}+\sigma^{\prime}{ }_{i}$. With the stress $\sigma_{i}$ expressed by equation 4 or 19 one finds the resultant stress,

$$
\begin{equation*}
\sigma^{\prime \prime}{ }_{i}=0.275(1+\mu) \frac{P}{h^{2}}\left[\log _{10}\left(\frac{E h^{3}}{k \cdot b^{4}}\right)-54.54\left(\frac{l}{L}\right)^{2} Z\right] . \tag{.36}
\end{equation*}
$$

in terms of $k$, or

$$
\begin{align*}
\sigma^{\prime \prime}{ }_{i}= & 1.1(1+\mu) \frac{P}{h^{2}}\left[\log _{10}\left(\frac{h}{b}\right)+\frac{1}{3} \log _{10}\left(\frac{E}{K}\right)\right. \\
& \left.-0.089-13.64\left(\frac{l}{L}\right)^{2} Z\right] \ldots \tag{37}
\end{align*}
$$

in terms of $K$.
In interpreting the results of tests the new equations may be used as follows: Values of $Z$ and $\frac{L}{l}$ are estimated (for example, $\frac{L}{l}=5$ ). The diagram for $z^{\prime \prime}=z+$ $z^{\prime}$ is drawn by use of Figure 4 in the previous paper and by use of equations 22 and 33 . With $Z$ and $\frac{L}{l}$ chosen within proper ranges this diagram should correspond in shape to those obtained experimentally for a series of points. A comparison of the experimental and theoretical diagrams of deflections for a series of points will lead to a determination of $D$ and $l$. Then $E$ may be computed by equation 18 , and $k$ or $K$ by equation 11 or 12 , respectively. Finally the stress at the bottom of the slab may be computed by equation 36 or 37 , and
the computed values may be compared with those obtained experimentally. It is noted that with a chosen value of $\frac{L}{T}$ a variation of $Z$ will cause different proportional variations of the computed deflections (equation 34 ) and stresses (equation 36 or 37 ). After some adjustments of the estimated values of $Z$ and $L$ it should be possible to establish a fair agreement between the computed and observed values of deflections and stresses and the value of $E$ obtained by tests of cylinders or control beams.

## EXTREME CASE IN WHICH THE SUBGRADEIS ASSUMED TO ACT AS

 A SOLID WITH CONSTANT MODULUS OF ELASTICITYThe preceding theory may be applied in the study of this special case. The problem is to determine $L$ and $Z$ so that the deflections of the subgrade due to the load, $k z+q$, will agree fairly closely with the deflections, $z^{\prime \prime}$, of the slab. Except for the possibility of a local hearing of the subgrade directly under the load, the value of $Z$ determined in this way may be assumed to be the upper limit of $Z$. The case is of interest for this reason. The value $Z=0$ may be assumed to be a lower extreme, except for the possibility of a low or soft spot in the subgrade directly under the load.

An approximate analysis was made under the assumption that the subgrade has a constant modulus of elasticity in compression, $E_{s}$, and a constant Poisson's ratio, $\mu_{s}$. This analysis gave the results.

$$
\begin{equation*}
L=5 l, Z=0.3906, K=0.1242 \frac{E_{s}}{1-\mu_{s}^{2}} \ldots \tag{38}
\end{equation*}
$$

By substituting these values of $L$ and $Z$ in equations 34 and 37 one obtains, for the particular case,

$$
\begin{equation*}
z^{\prime \prime}{ }_{i}=0.07617 \frac{P}{D^{-}} \tag{39}
\end{equation*}
$$

and

$$
\begin{equation*}
\sigma^{\prime \prime}{ }_{i}=1.1(1+\mu) \frac{P}{h^{2}}\left[\log _{10}\left(\frac{h}{b}\right)+\frac{1}{3} \log _{10}\left(\frac{E}{K}\right)-0.302\right] \tag{40}
\end{equation*}
$$

The maximum reaction per unit of area was found to be $0.350 \frac{P}{l^{2}}$ instead of $0.125 \frac{P}{l^{2}}$ as obtained in the original theory.

Under actual conditions it must be expected that $0<Z<0.39$. Then equations 34 and 37 will apply instead of equations 39 and 40 .

## AN IMPROVED RECORDING STRAIN GAGE

Reported by L. W. TELLER, Senior Engineer of Tests, U.S. Bureau of Public Roads


Figure 1.--Early Type of Recording Strain Gage.

SELERAL years ago there was developed in the Division of Tests a small recording strain gage designed primarily for the determination of stresses in concrete structures. This gage, which has been described in detail elsewhere ${ }^{1}$ was about 6 inches in length, of small cross section, employed a single bell-crank lever giving a multiplication ratio of about 60 to 1 , and recorded on a smoked glass slide. It is shown in figure 1.
Subsequent to its development, extensive use was made of the instrument in a number of research projects dealing with stress conditions in concrete structures. ${ }^{2}$
Experience with the gage on the projects mentioned demonstrated the value of a strain gage of this size and type and indicated certain defects in the existing design, the elimination of which would improve both the quality of the data obtained and the mechanical functioning of the instrument. For example, the original gage was built with the gage body of brass, a metal poorly suited to the purpose because its high conductive and ligh expansive properties made the gage very sensitive to thermal changes. Mechanically there were difficulties with the manipulation of the slide on which the record was made and with the functioning of the fulcrum of the bell-crank lever. Fundamentally, the principle of the gage appeared to be satisfactory and with the idea of overcoming the temperature and mechanical troubles mentioned above the instrument was completely redesigned, retaining the principle of the original gage but introducing new materials and new mechanical features wherever it appeared that these would improve the functioning of the instrument.

[^3]Several gages were built according to the new design and these have now been in use for more than a year. Careful calibration tests and a very considerable amount of field work with these improved gages indicate that the objects sought in the redesign have been largely attained. Since a satisfactory gage of this type can be put to many uses, it was thought that a complete description would be of general interest.

## temperature effects important in design of strain gage

Changes in temperature cause dimensional changes in most materials, and probably the most serious difficulty usually encountered in making strain measurements is the separation of the linear changes due to stress from those which are caused by thermal changes either in the material being stressed or in the gage itself. In the case of measurements in concrete, the thermal changes in the material take place rather slowly because of the low thermal conductivity of the concrete, and the dimensional changes in the material occur at a correspondingly low rate. Unless the stressing of the concrete is extended over a period of time sufficient to permit the concrete to change in temperature by about one degree centigrade or more the effect of temperature on the material does not ordinarily have to be taken into account. The rapidity with which the metal in the gage itself responds to thermal changes makes the effect of such changes on the dimensions of the gage an important consideration in the design, the ideal design being one in which complete compensation for the effect of changes in gage temperature is provided.
As previously noted the original gages were built with bodies of brass. This metal is highly conductive and has relatively a high coefficient of thermal expansion (about 0.000019 per degree centigrade). The result is that a gage built of brass would change temperature very quickly and the changes in the length of the gage would be relatively large. For example, if one assumes that the modulus of elasticity for concrete is $4,000,000$ pounds per square inch it is apparent that the elongation of a brass gage due to a temperature rise of $1^{\circ} \mathrm{C}$. Would equal the elongation of the concrete due


Figure 2.-Improved Type of Recording Strain Gage.


NOTE
THE DIRECTION OF MOTION OF THE VARIOUS
PARTS OF THE SYSTEM IS INDICATED BY THE
ARROWS
Figure 3.- Diagram Showing the Mechanical Functioning of the Gage.
to a tensile stress of 76 pounds per square inch. This is approximately one fifth to one fourth of the ultimate strength in tension of the concrete; and when one considers that variations of several degrees in the temperature of the air surrounding the strain gage are not uncommon, the seriousness of this temperature effect is evident.

DESCRIPTION OF IMPROVED GAGE
Three riews of the redesigned instrument are shown in figure 2. Figure 3 is a diagram illustrating the manner in which the gage functions, and figure 4 is a "phantom" sketch showing in perspective the relative positions of the essential parts of the instrument.

The gage consists of three major parts-a gage body (A) on which is mounted a bell-crank lever (B) and a slide carriage (C). In one end of the gage body there is a freely sliding pin (D), the outer end of which is conical and rests in one of the gage points, while the inner end is a plane surface which bears against the knife edge of the stud or post (E). The latter is fastened rigidly to the bell-crank lever and moves freely in the oversize hole in the gage body. The distance from this knife edge to the fulcrum plate or spring hinge (F) constitutes the short arm of the bell-crank lever, while the long arm is formed by the two metal rods (G) carrying the stylus point (H). This stylus point bears against the smoked surface of a small glass plate (I) mounted in the slide carriage. After each record is made this carriage is moved along the body of the gage
by rotating the capstan nut (J) on the threaded portion of the gage body. An adjustable conical point (K) in the end of the gage body nearest to the slide carriage rests in the other gage point.

## OPERATION OF GAGE DESCRIBED

In use the gage is placed between the gage points as shown in figure 5 and the length is adjusted until the ends of the gage rest firmly in the gage points and the stylus point is in a position about half-way across the glass slide. In this position either tensile or compressive deformations can be measured.

When a change in the distance between the gage point occurs, the sliding pin D moves longitudimally in the gage body and this translation causes a rotation of the stud ( E ) about the fulcrum. This stud being a part of the bell-crank lever, a corresponding motion of the stylus point across the glass slide occurs, the magnitude of the two displacements being in the ratio of the lengths of the respective arms of the bell crank lever, or about 60 to 1 . This movement of the stylus point produces a trace in the smoke film on the glass and the length of this trace obviously bears a direct relation to the change in distance between the gage points. After the recording of the deformation has been completed the capstan nut ( J ) is rotated, moving the slide carriage longitudinally to a new position.

Twenty or more records can be made on one glass slide. The appearance of two typical completed records on the glass slide is shown in figure 6 .


Figure 4.-Phantom Shetcif, Showing Relative Positions of Working Parts of Gage.


Figure 5.-Tmproved Type of Recordíg Stran Gage Mounted between Gage Points. View Looking Down on Top of Instrument.


Figure 6.-Completed Strain-Gage Records. These Slides Hate Been Fixed and Are Ready for Measurement of the Length of the Displacemexts in the Direction Perpendicular to the Long Axis of the Síide.

## COMPENSATION FOR THERMAL EXPANSION DEVISED

In designing the gage it was decided to reduce the thermal expansion to a minimum through the use of the steel-nickel alloy known as "Inv"ar." This material has a guaranteed coefficient of thermal expansion of not to exceed $0.8 \times 10^{-6}$ (or about 4 percent of that of brass). The entire gage body from one gage point to the other is made of this metal. But even this
material changes in dimension when subjected to temperature changes; and in order to compensate within the gage for such changes the stylus arm was designed with two metal rods having slightly different coefficients of thermal expansion and so spaced as to curve the stylus arm to an extent sufficient to compensate for the rotation of the bell crank caused by the expansion or contraction of the gage parts. While it would probably have been possible to compute the length and spacing of these rods, it was found better in this instance to adjust these dimensions by trial.

To do this the frame shown in figure 7 was built A steel having a known coefficient of thermal expansion was used for the purpose. Is will be noted in the photograph the side members are hollow and contain two mercurial thermoneters. The gage was mounted hetween the cross members of the frame. I wooden box which provided a support for the frame and served to maintain a uniform temperature around it completed the apparatus.

With the strain gage in the frame, the whole was subjected to changes in temperature greater than would be encountered in the normal operation of the gage and from the data obtained the error due to gage


Figure 7.-Steel Frame used for Determining the Effect of Cianges in the Temperature of the Strain Gage and for Adjusting the Bimetallic Arm.
expansion was determined. The compensating feature of the stylus arm was adjusted, by trial, until the length of the trace in the smoke film on the strain-gage slide was that corresponding to a change in gage length equal to the expansion (or contraction) in the steel frame.

While perfection was not attained in this adjustment the compensation feature has reduced the error arising from temperature changes in the gage to a point where it is negligible for the usual operating conditions under which the gage is used.

## OTHER IMPROVEMENTS MADE

As will be noted by reference to figures 2, 3, and 4 the bell-crank lever operates about a plate-fulcrum or spring hinge. The substitution of steel for brass in the supporting parts for this steel spring hinge apparently has eliminated a trouble which was frequently had with the original design, caused by the separation of the spring from the supporting metal near the fulcrum point. Such separation resulted in a slight shifting of the flexure point of the spring and a corresponding change in the magnification ratio of the gage.

The slide carriage used on this gage is new. The glass slide is mounted between bronze spring clips which are fastened to the sides of the slide carriage. The carriage itself slides orer the threaded portion of the gage body, being kept from rotating by a spline which slides in a keyway in the body of the gage. This is shown clearly in figure ?. A capstan nut threaded on the gage body operates in a rectangular notch in the extended end of this spline. Rotation of the mut moves
it along the gage and draws with it the spline and the slide carriage to which the spline is attached.

## CALIBRATIONS MADE ON SPECIAL MACHINE

For the calibration of these gages and similar linear measuring instruments, the special calibrating device shown in figure 8 was designed and built.

It consists of a cast-iron base carrying three pedestals. In the pedestal at the right (see fig. 8) there is a stationary cylindrical steel block which may be adjusted horizontally to any desired position and clamped. In the pedestal at the left is a long nut into which is threaded a precise micrometer screw. This screw is provided with a graduated drum reading directly to 0.0001 inch and a knurled operating knob at its outer end. Its inner end is finished with a plane surface of hardened steel. The combination of screw, nut, and graduated drum was calibrated for accuracy of displacement indication by the National Bureau of Standards and the errors determined to the nearest one hundred thousandth of an inch.

In the center pedestal is a movable steel block of triangular cross section which slides longitudinally in $\checkmark$ bearings under the action of the micrometer screw. The movable block is fitted with a spherical surface where it makes contact with the end of the screw: This block is kept seated in its $V$ guides by springpressed rollers which bear on its upper surface. A firm contact between the block and the micrometer screw is maintained by means of a suspended weight which exerts a longitudinal pull on the movable block through a flexible ribbon operating over a pulley.

In use the gage to be calibrated is mounted between the fixed block and the movable block as shown in figure 8. The movable block is then displaced horizontally definite amounts by means of the micrometer screw, the amount of the displacement being determined by the graduations on the drum. After each displacement the glass record slide on the strain gage is mored ahead to a new position. It has been customary to use displacement increments of five ten-thousandths of an inch, to repeat each setting several times, and to have each gage calibrated independently by two operators.

The glass slide on which the record is made is coated with a light smoke film. After the record is complete and such identifying numbers as are desired have been marked in the film, the record is fixed by a coating of copal gum dissolved in acetone. These records (see fig. 6) may be measured satisfactorily in either one of two ways. The length of the trace scribed by the stylus point may be measured directly with a measuring microscope or a linear comparator, or the glass slide may be mounted in a projection apparatus and an enlarged image thrown on a screen containing rectangular coordinates by means of which the length of the projected trace is measured. While both methods have been used in the work of the Bureau of Public Roads, the latter method has been found more convenient. For this work a projection apparatus was built, the magnifying device being an ordinary laboratory microscope. This projector gives a magnification of approximately $30: 1$, which, combined with the gage magnification of about $60: 1$, gives a total magnification of about eighteen hundred times the original displacement.

Figure 9 is the calibration curve of one of these grages, the true displacement and the length of the


Figure 8.-Special Calibrating Device for the Application and Precise Measurement of Smale Linear Displacements to Strain Gages, Micrometer Dials, And Similar Apparatus.
projected record on the strain gage slide being used as the coordinates. The multiplication ratio is constant and for this particular gage is shown to be slightly less than 58:1.

## PERFORMANCE CHARACTERISTICS ARE SATISFACTORY

The accuracy of strain determination possible with this gage depends principally on the accuracy with which the travel of the stylus point in the smoke film can be measured. This in turn depends upon the sharpness of the record left by the stylus. The smoke film is a layer of carbon which is scraped aside by the stylus as it moves across the slide. Some of the carbon piles up ahead of the stylus and remains in a minute heap at the end of the trace. The blunter the stylus point and the thicker the smoke film the broader will be the record, the larger will be this heap, and the more difficult will be the determination of the true distance traveled by the point. Hence, the accuracy of the measurement can be considerably increased if the stylus point is kept sharp and if the smoke film on the glass slide is made as thin as possible.

If these two points are given attention the record will be clearly defined and its length, when projected on the screen, can be measured with certainty to the nearest 0.05 inch. Then, since the gage length is about 6 inches and the combined magnification of the strain gage lever and of the projection apparatus is approximately $1800: 1$, the least unit deformation which could be measured definitely would be $\frac{0.05}{1800 \times 6}$ or 0.0000046 inch per inch.

In concrete, if we assume a modulus of elasticity of $4,000,000$ pounds per square inch, this deformation corresponds to a unit stress of 18 pounds per square inch. In the work which the Bureau has done with these grages it has been considered that the stress deter-


Figure 9.-Calibration Curve of a Recording Strain Gage of the Improved Type.
minations were prohably accurate to 20 or 25 pounds per square inch.

The consistency of performance can best be illustrated by the following table, which shows the measured length of projected record for 24 repetitions of a total displacement of 0.0010 inch.

The arerage of the 24 measured lengths is 2.015 inches. The maximum deviation is 5.7 percent, the
average deviation is 1.6 percent, and only 3 observations (tests nos. 3, 15, and 21) deviate more than 2 percent from the mean of the group of 24 .
${ }^{[ } p$ to the present time the strain gages have been used entirely for the measurement of strains induced in concrete parement slabs by slowly applied loads and no study has been made of their behavior when subjeeted to the sudden deformations caused by impact loads. It is plamned to make such a study later. Inder the conditions noted, many thousands of measurements have been made and the behavior of the gages throughout the work has been highly satisfactory.

## (Continued from page 184)

## SUMMARY

In conclusion, attention is called to certain practical considerations of the problem as regards (a) the moisture content required to furnish the desired fluidity and $(b)$ the definition of detrimental shrinkage. From the foregoing discussion it may be inferred that the desired consistency of mix is furnished by only one moisture content which must be determined with a high degree of exactness. It seems likely that pastes having the low consistency required in mud-jack work may be equally suitable with moisture contents varying within a range of 5 percent or more.

A water content of 42 percent has been suggested for the mixture using soil $\mathrm{S}-6361$ but in practice the consistency of mix is determined by the operator on the hasis of visual inspection. Successive batches probably had moisture contents varying from about 40 to 45 or 46 percent and all appeared to work equally well. An experienced laboratory worker, on the basis of risual inspection, would not be able to determine more accurately the moisture required to furnish equal consistency in several mixtures made from the same soil.

If the flow curve can be used to indicate the required moisture content within possibly 5 percent, its use for the purpose proposed is warranted. Whether the optimum water content for all soils is that corresponding to 0.1 blow, as was the case for soil S-6361 must be determined by additional investigation.

It is known that detrimental shrinkage cannot occur when the shrinkage limit is equal to or greater than the moisture content of the mixture. It is not known how much the shrinkage limit can be below this moisture content and still aroid harmful shrinkage.

A 7 -percent portland cement admixture completely eliminated the possibility of shrinkage of paste made with soil S-6361 and this is desirable whenever it can be accomplished. Is a matter of fact, had this paste contained as much as 49 percent of moisture it would not have shrunk on loss of moisture since the shrinkage limit of the mixture with 7 percent of cement was 49. Howerer, soils of the excellent character of S-6361 are not arailable in many parts of the United States. In some cases it may be impractical to add enough cement to the soils which must be used, to completely eliminate

Table 1.-Calibration data for gage no. 8; true displacement 0.0010 inch

| Test no. | Length of record | Test no. | Length of record | Test no. | Length of record |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Inches |  | Inches |  | Inches |
|  | 2. 000 | 4. | 2. 000 | 17 | 2. 000 |
|  | 2. 050 | 10 | 2. 025 | 18. | 2.050 |
|  | 1. 9.50 | 11 | 2. 025 | 19 | 2. 050 |
|  | 2. 000 | 12 | 2. 025 | 20. | 2. 000 |
|  | 2. 050 | 13 | 2. 050 | 21 | 1. 925 |
|  | 2. 050 | 14 | 2. 050 | 22 | 2. 000 |
|  | 2. 050 | 15 | 1. 900 | 23 | 2.050) |
|  | 2. 050 | 16 | 2. 000 | 24 | 2. 000 |

all possibility of shrinkage. However, a conception of the basic physical requirements of pastes for use in mud jacking and a knowledge of the simple laboratory tests for disclosing the properties of soils and soil mixtures will assist materially in the selection of the best soils at hand.

A laboratory investigation of this character should include the following steps:

1. Hydrometer method of analysis to determine the grading.
2. Determination of the flow curve of the soil without admixture to furnish the liquid limit and give some idea of the moisture content at the desired consistency of mix.
3. Determination of the shrinkage limit corresponding to several admixtures of cement to determine the effect of cement in reducing the shrinkage of the mix on loss of moisture.

For all practical purposes it can be assumed that the mixture with the cement admixture will require several percent more moisture than that of the soil and water without the admixture.

The possibilities of soil admixtures for improving the grading of soils should not be overlooked. Other materials which have been suggested as admixtures in soil stabilization ${ }^{3}$ may be found useful.

It may be found impractical to eliminate all shrinkage by admixture of portland cement and possibly fine sand but the tests will show which mixtures can be most easily improved by such means and consequently which soils of a group are apt to make the best mixtures.

It is safe to assume that all soils which the suggested laboratory tests show to have the same characteristics as soil S-6361 are physically the same and therefore should duplicate the good performance of this soil in mud jacking. We are not in a position, however, to say how far the characteristics of soils may vary from those of S-6361 and still be suitable for mud jacking.

The four soils described in this report illustrate the great variation which may exist in the character of soils selected for mud jacking by visual inspection on the basis of our present knowledge of the subject. They emphasize the need for additional information regarding the character of such soils and tests for properly identifying them.

[^4]AS PROVIDED IN TITLE II, SECTION 204 OF THE NATIONAL INDUSTRIAL RECOVERY ACT
CLASS III-PROJECTS ON SECONDARY OR FEEDER ROADS
AS OF NOVEMBER 30,1933

| State | PUBLIC WORKSFUNDS ASSIGNEDFOR ALASS IINPPRJETSONSECONDARYHIGHWAYS | COMPLETED |  |  | UNDER CONSTRUCTION |  |  |  | APPROVED FOR CONSTRUCTION |  | BALANCE OF FUNDS AVAILABLE FOR NEW PROJECTS | state |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | al cost | Public works funds | Mileage | ated | Public works funds | Percentage compieted | Milea | Public works funds atloted | Mileage |  |  |
|  | ,092 | \$ | \$ |  | \$ | \$ |  |  | 85,000. | 13.5 |  | Alaba |
| Arizona Arkansas | 625.435 1.687 .084 |  |  |  | 40.700.00 | 40,700.00 |  | 6.0 | 88, 098. 85 | 1.7 | $\begin{array}{r} 584,735.00 \\ 1.598 .985 .15 \end{array}$ | Arizona Arkansas |
| California Colorado. Connecticut | $\begin{array}{r} 3.901 .838 \\ 1.718,632 \\ 659.120 \end{array}$ |  |  |  | $\begin{aligned} & 637.625 .93 \\ & 689.262 .42 \\ & 211.882 .58 \end{aligned}$ | $\begin{aligned} & 499.573 .95 \\ & 689.262 .42 \end{aligned}$ $211.382 .58$ | $\begin{array}{r} 7.2 \\ 15.5 \end{array}$ | $\begin{array}{r} 26.0 \\ 139.9 \\ 3.1 \end{array}$ | $\begin{aligned} & 763.429 .57 \\ & 185,000.00 \end{aligned}$ | $\begin{aligned} & 73.0 \\ & 41.9 \end{aligned}$ | $\begin{array}{r} 2,638,834.48 \\ 844,369.58 \\ 447,237.42 \end{array}$ | California Colorado Connecticut |
| Delaware Florida Georgia | $\begin{array}{r} 454,712 \\ 1,307.958 \\ 2,320.973 \end{array}$ |  |  |  | 500, 816.33 | 500,516.33 | 3.0 | 25.6 | $\begin{array}{r} 576,073.59 \\ 76,055.21 \end{array}$ | 42.8 6.8 | 454.712.00 <br> 231.068.08 <br> 2,244,917.79 | Delaware Florida Georgi |
| Idaho Illinois Indiana | $\begin{array}{r} 1,121,562 \\ 6,262,223 \\ 501,892 \end{array}$ | 11,625.34 | 7.523.08 | 2.8 | 540,602.96 250,311.61 77,000.00 | $\begin{array}{r} 540,602.96 \\ 250,31.61 \\ \pi .000 .00 \end{array}$ | 30.0 4.0 | $\begin{array}{r} 103.1 \\ 29.9 \\ 19.1 \end{array}$ | $\begin{array}{r} 94,358.09 \\ 1.536,124,68 \\ 88,000.00 \\ \hline 8 \end{array}$ | 10.5 128.7 29.0 | $\begin{array}{r} 479.077 .87 \\ 4.475 .786 .71 \\ 336,892.00 \end{array}$ | Idaho Illinois Indiana |
| Iowa Kansas Kentucky | $\begin{aligned} & 2,212,245 \\ & 2,522,401 \\ & 1,879,340 \end{aligned}$ | 28,117.41 | 26,600.00 | 4.6 | 532.025 .44 403.334 .21 | $\begin{array}{r} 507.300 .00 \\ 10.651 .49 \end{array}$ $403.334 .21$ | 19.3 .5 | $\begin{aligned} & 37.8 \\ & 3.4 \\ & 33.3 \end{aligned}$ | $\begin{array}{r} 164,600.00 \\ 826,662.38 \\ 1.360,915.02 \end{array}$ | $\begin{array}{r} 14.4 \\ 62.8 \\ 180.1 \end{array}$ | 1.513 .745 .00 <br> 1,685.087.13 <br> 115.089.99 | Iowa <br> Kansas <br> Kentucky |
| Louisiana Maine Maryland | $\begin{array}{r} 1.457,148 \\ 842,479 \\ 891,132 \end{array}$ | 186.151.34 | 186,151.34 | 23.2 | 59.390 .34 673.972 .74 | $\begin{array}{r} 59,390.34 \\ 629,407.73 \end{array}$ | $\begin{aligned} & 40.4 \\ & 87.2 \end{aligned}$ | $\begin{array}{r} 2.3 \\ 7.7 \end{array}$ | $365,389,88$ $26,634.85$ 115.754. 25 | $\begin{array}{r} 14.4 \\ i .2 \\ i .7 \end{array}$ | $\begin{array}{r} 1,032,367.78 \\ 285.08 \\ -775.377 .75 \\ \hline \end{array}$ | Louisiana Maine Maryland |
| Massachusetts Michigan. Minnesota | $\begin{array}{r} 522,768 \\ 3.144,057 \\ 2.131 .314 \end{array}$ |  |  |  | $\begin{aligned} & 367.127 .97 \\ & 553.250 .90 \\ & 904.177 .04 \end{aligned}$ | $\begin{aligned} & 367.127 .97 \\ & 553,250.00 \\ & 896.429 .91 \end{aligned}$ | $\begin{aligned} & 39.0 \\ & 17.5 \end{aligned}$ | $\begin{array}{r} 11.5 \\ 24.0 \\ 1461.9 \end{array}$ | $\begin{array}{r} 1.236,350.00 \\ 604.936 .14 \end{array}$ | $\begin{aligned} & 120.1 \\ & 95.7 \end{aligned}$ | $\begin{array}{r} 160,640.03 \\ 1,394,457.00 \\ 1.39 .947 .95 \end{array}$ | Massachusetts Michigan Minnesota |
| Mississippi Missouri Montana | $\begin{aligned} & 1.744,669 \\ & 3.045,076 \\ & 1.859,937 \end{aligned}$ |  |  |  | $\begin{aligned} & 435,000.00 \\ & 702,288.32 \\ & 120.587 .26 \end{aligned}$ | $\begin{aligned} & 435.000 .00 \\ & 702,288.32 \\ & 120,587.26 \end{aligned}$ | $\begin{gathered} \begin{array}{c} 10.8 \\ 7.5 \\ 2.5 \end{array} \end{gathered}$ | $\begin{array}{r} 54.6 \\ 134.2 \\ 4.4 \end{array}$ | $754,102.18$ <br> 355.941.47 | $\begin{gathered} 132.8 \\ 50.4 \end{gathered}$ | $\begin{aligned} & 1.309,669.00 \\ & 1.588,685.50 \\ & 1.383 .408 .27 \end{aligned}$ | Mississipp Missouri Montana |
| Nebraska Nevada New Hampshire | $\begin{array}{r} 1,957,240 \\ 1,136,479 \\ 47,460 \end{array}$ |  |  |  | $\begin{array}{r} 1.10,1,827.06 \\ 358,863.14 \\ 234.358 .45 \end{array}$ | $\begin{array}{r} 1,099,964.74 \\ 338,863.14 \\ 234,558.45 \end{array}$ | $\begin{array}{r} 5.5 \\ 13.6 \\ 29.0 \end{array}$ | $\begin{array}{r} 238.9 \\ 52.4 \\ 9.7 \end{array}$ | $126,850.25$ 99.882.19 | $\begin{gathered} 21.8 \\ 14.9 \\ 6.7 \end{gathered}$ | 730.425 .01 143.219 .36 | Nebraska Nevada New Hampshire |
| New Jersey New Mexico New York | $\begin{array}{r} 63,460 \\ 1,448,234 \\ 3,662,137 \end{array}$ | 6,300.00 | 6,300.00 | .1 | $\begin{array}{r} 522.500 .00 \\ 3.082,363.19 \end{array}$ | $\begin{array}{r} 522,500.00 \\ 2,884,063.19 \end{array}$ | $\begin{array}{r} 1.7 \\ 15.1 \end{array}$ | 250.8 73.2 | 56.550 .52 256.000 .00 767. 300.00 | $\begin{array}{r} 225.5 \\ 19.1 \end{array}$ | $\begin{array}{r} 6.909 .48 \\ 669,734.00 \\ 4,473.81 \end{array}$ | New Jersey New Mexico New York |
| North Carolina North Dakota Ohio | 2,380,573 <br> 1,451,112 <br> 3.871,148 |  |  |  | $\begin{aligned} & 507.308 .33 \\ & 793.500 .00 \end{aligned}$ | $\begin{aligned} & 502,026.56 \\ & 688,004.90 \end{aligned}$ | 16.3 13.1 | 40.0 180.4 | $\begin{array}{r} 364,509.77 \\ 46,035.01 \\ 2.213 .230 .00 \end{array}$ | $\begin{aligned} & 30.5 \\ & 10.4 \\ & 75.7 \end{aligned}$ | $\begin{array}{r} 1,514,044.67 \\ 1,405.076 .99 \\ 969.853 .10 \end{array}$ | North Carolina North Dakota Ohio |
| Oklahoma Oregon Pennsylvania | $\begin{aligned} & 2.304,199 \\ & 1.526,124 \\ & 7.716 .975 \end{aligned}$ |  |  |  | $\begin{array}{r} 168,868.15 \\ 4,500,948.76 \end{array}$ | $\begin{array}{r} 168,868.15 \\ 4.500,948.76 \\ \hline \end{array}$ | $11.2$ | 77.2 478.4 | $\begin{array}{r} 41,159.13 \\ 532,412.39 \\ 2,131,774.16 \end{array}$ | $\begin{array}{r} .7 \\ 60.6 \\ 170.8 \end{array}$ | $\begin{aligned} & 2,263,039.87 \\ & 82.443 .46 \\ & 1,084,652.46 \end{aligned}$ | Oklahoma Oregon Pennsylvania |
| Rhode Island South Carolina South Dakota | $\begin{array}{r} 499,677 \\ 1.364 .791 \\ 1.502,870 \end{array}$ |  |  |  | $\begin{array}{r} 766.612 .29 \\ 80,636.49 \end{array}$ | $\begin{array}{r} 766,612.29 \\ 80,636.49 \end{array}$ | $\begin{array}{r} 2.1 \\ 38.4 \end{array}$ | $\begin{aligned} & 96.9 \\ & 30.9 \end{aligned}$ | $\begin{array}{r} 9.889 .00 \\ 204,216.54 \end{array}$ | 1.1 25.0 | $\begin{array}{r} 489,788.00 \\ 393,962.17 \\ 1.422,233.51 \end{array}$ | Rhode Island South Carolina South Dakota |
| Tennessee Texas. Utah | $\begin{aligned} & 2,123,155 \\ & 6,064,006 \\ & 1,048.677 \end{aligned}$ | 61.543.36 | 61.543 .36 | 8.0 | $\begin{array}{r} 87.972 .50 \\ 1.472 .019 .59 \\ 457.339 .35 \end{array}$ | $\begin{array}{r} 87.972 .50 \\ 1.106 .072 .58 \\ 457,339.35 \end{array}$ | 9.9 2.6 | $\begin{array}{r} 8.5 \\ 210.5 \\ 122.3 \end{array}$ | $\begin{array}{r} 463,303.83 \\ 1.483,98.00 \\ 191.342 .66 \end{array}$ | 52.5 308.9 8.9 | $\begin{array}{r} 1,571,878.67 \\ 3,471,005.42 \\ 338,451.63 \end{array}$ | Tennessec <br> Texas <br> Utah |
| Vermont Virginia Washington | $\begin{array}{r} 465,026 \\ 1,854,189 \\ 1,180,362 \end{array}$ |  |  |  | $241,426.64$ <br> 815.399 .51 449.857 .95 | $\begin{aligned} & 232,346.12 \\ & 815.399 .51 \\ & 449,857.35 \end{aligned}$ | $\begin{array}{r} 6.0 \\ 17.4 \\ 15.8 \end{array}$ | $\begin{array}{r} 20.3 \\ 100.2 \\ 34.6 \end{array}$ | $\begin{array}{r} 94,931.88 \\ 246,298.59 \\ 203.576 .55 \end{array}$ | $\begin{array}{r} 8.9 \\ 22.8 \\ 10.6 \end{array}$ | 137.748 .00 792.490 .90 526.928.10 | Vermont <br> Virginia <br> Washington |
| West Virginia <br> Wisconsin <br> Wyoming | $\begin{aligned} & 1,118.559 \\ & 2,431.20 \\ & 1.125 .332 \end{aligned}$ |  |  |  | $\begin{array}{r} 83.064 .57 \\ 1.180 .135 .41 \\ 461.759 .26 \end{array}$ | $\begin{array}{r} 83.064 .57 \\ 1.141 .120 .13 \\ 461.759 .26 \end{array}$ | $\begin{array}{r} 3.6 \\ 19.5 \\ 6.9 \end{array}$ | $\begin{array}{r} 2.8 \\ 83.2 \\ 79.8 \end{array}$ | 147.135 .19 113.966 .04 | $\begin{array}{r} 9.4 \\ 13.5 \end{array}$ | $\begin{array}{r} 888,359.24 \\ 1.192,961.39 \\ 549,606.10 \end{array}$ | West Virginia <br> Wisconsin <br> Wyoming |
| District of Columbia Hawaii | $\begin{aligned} & 767,388 \\ & 187,106 \end{aligned}$ |  |  |  | 330.901 .12 | 330.901.12 | 24.1 | 3.4 | 385.739. 86 | 2.1 | $\begin{array}{r} 50.747 .02 \\ 187.106 .00 \end{array}$ | District of Columbia Hawaii |
| TOTALS | 94,676,687 | 293.737 .45 | 288,117.78 | 38.7 | 25.407.668.40 | $24.447,656.24$ | 14.5 | 3.017 .7 | 19.781,272.72 | 2,127.4 | 50,159,640.26 | TOTALS |


SUMMARY OF CLASSES I, II, AND III

| STATE | $\begin{aligned} & \text { TOTAL } \\ & \text { APPORTIONMENT } \\ & \text { OF } \\ & \text { PUBLIC WORKS } \\ & \text { FUNDS } \end{aligned}$ | COMPLETED |  |  |  | UNDER CONSTRUCTION |  |  |  |  | APPROVED FOR CONSTRUCTION |  | BALANCE OF PUBLIC WORKS FUNDS AVAILABLE FOR ALL NEW PROJECTS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Total cost | Public works funds | $\underset{\substack{\text { Regular } \\ \text { aid }}}{\text { Federal }}$ | Mileage | Estimated total cost | Public works funds | Regular Federal aid allotted | Percentage <br> completed | Milcag | $\underset{\substack{\text { Public works funds } \\ \text { allotted }}}{ }$ | Mileage |  |
| Alabama Arizona Arkansas | 8, 370,133 <br> 5.211.960 <br> 6,748,335 | \$ | \$ | \$ |  | 2,996,456.19 <br> 2.447,665.92 989, 890.89 | $\begin{array}{r} \$, 660,289.19 \\ 2.477 .665 .92 \\ 667,420.55 \end{array}$ | $\$ 1,334,167.00$ $322,470.34$ | $\begin{array}{r} 8.4 \\ 15.2 \\ 12.4 \end{array}$ | $\begin{array}{r} 142.7 \\ 174.6 \\ -\quad 23.8 \\ \hline \end{array}$ | $\begin{gathered} \$ 1.092,600.55 \\ 309.079 .55 \\ 666.438 .77 \end{gathered}$ | $\begin{array}{r} 123.7 \\ 2.9 \\ 4.9 \\ \hline 4.4 \end{array}$ | $\begin{aligned} & \$ .617,243.26 \\ & 2.455 .214 .53 \\ & 2.414 .425 .68 \end{aligned}$ |
| California Colorado Connecticut | $\begin{array}{r} 15.607 .354 \\ 6.374 .530 \\ 2.865 .740 \end{array}$ | 132,258.17 | 132,258.17 |  | 5.1 | $\begin{aligned} & 6.769,41,61 \\ & 2,188,158.30 \\ & 1,056,975.87 \end{aligned}$ | $\begin{aligned} & 5.658,509.28 \\ & 2,188,158.30 \\ & 1,056.975 .87 \end{aligned}$ |  | $\begin{aligned} & 9.6 \\ & 31.4 \\ & 13.5 \end{aligned}$ | $\begin{aligned} & 212.4 \\ & 266.7 \\ & 11.3 \end{aligned}$ | $\begin{array}{r} 2.4544 .347 .34 \\ 1.934 .551 .60 \\ 1.091 .170 .93 \end{array}$ | $\begin{array}{r} 134.1 \\ 67.9 \\ 19.5 \end{array}$ | $\begin{array}{r} 7.494 .497 .38 \\ 3.619 .561 .93 \\ 717.593 .20 \end{array}$ |
| Delaware <br> Fiorida Georgia | 1,519,088 10.231 .834 10.091 .185 |  |  |  |  | $\begin{array}{r} 673.933 .20 \\ 2,963,558.27 \\ 363.200 .66 \end{array}$ | $\begin{array}{r} 673.933 .20 \\ \text { 2.044.0.044.54 } \\ 363,200.66 \end{array}$ | 921,943.33 | $\begin{aligned} & 37.5 \\ & 10.2 \\ & 10.6 \\ & 14.6 \end{aligned}$ | $\begin{array}{r} 24.4 \\ 108.5 \\ 14.7 \end{array}$ | $\begin{array}{r} 456.536 .30 \\ 885,944.42 \\ 1,224,292,30 \end{array}$ | $\begin{aligned} & 12.7 \\ & 50.5 \\ & 58.4 \end{aligned}$ | $\begin{array}{r} 688,618.50 \\ 2,304,884.74 \\ 8,503,691.54 \end{array}$ |
| Idaho Illinois Indiana | 4.486,249 10,037.843 | 64,589.98 | 59,357. 80 |  | 22.3 | 2,305,557.51 <br> 1,362,138.00 <br> 1,988, 204.53 | $\begin{array}{r} 2,290,924.76 \\ 1,362,138.02 \\ 1,988,204.83 \end{array}$ |  | $\begin{array}{r} 34.2 \\ 3.9 \\ 37.2 \\ \hline 17.2 \end{array}$ | $\begin{array}{r} 29.5 \\ 47.8 \\ 86.0 \end{array}$ | $\begin{array}{r} 334.360 .09 \\ 5.613 .110 .79 \\ 1,199.981 .89 \end{array}$ | $\begin{array}{r} 18.5 \\ 172.1 \\ \text { 172.1 } \\ 59.5 \end{array}$ | $\begin{array}{r} 1,301,606.35 \\ 10,595,521.19 \\ 6,849,656.28 \end{array}$ |
| Iowa <br> Kansas <br> Kentucky | 10.055 .660 $7.517 .359$ | 821.647 .87 | 769.150 .00 |  | 33.7 | $\begin{aligned} & 5,092.476 .46 \\ & 1.916,589.48 \\ & 1.597 .739 .20 \end{aligned}$ | $\begin{aligned} & 4.801,800.00 \\ & 1.916 .589 .18 \\ & 1.897 .739 .20 \end{aligned}$ |  | $\begin{aligned} & \begin{array}{l} 29.1 \\ 19.0 \\ 10.5 \end{array} \end{aligned}$ | $\begin{aligned} & 236.5 \\ & 200.0 \\ & 146.7 \end{aligned}$ | 1.345 .030 .00 <br> 3.911 .541 .24 <br> 2.510 .592 .63 | $\begin{array}{r} 84.5 \\ 279.3 \\ 247.0 \end{array}$ | $\begin{aligned} & 3.139,680.00 \\ & 4,261,473.58 \\ & 3.199 .027 .17 \end{aligned}$ |
| Louisiana <br> Maine <br> Maryland | $\begin{aligned} & 5.828 .591 \\ & 3.369 .917 \\ & 3.564 .587 \end{aligned}$ | 226,018. 23 | 226.018.23 |  | 24.0 | $\begin{array}{r} 873,847.98 \\ 2,028,003.30 \\ 173.094 .69 \end{array}$ | $\begin{array}{r} 873,847.98 \\ 1.978 .027 .18 \\ 159.295 .94 \end{array}$ |  | $\begin{aligned} & 39.1 \\ & 51.6 \\ & 15.7 \end{aligned}$ | $\begin{array}{r} 27.3 \\ 110.7 \\ 5.4 \end{array}$ | $\begin{array}{r} 1,703,657.67 \\ 508,108.81 \\ 790,368.20 \end{array}$ | $\begin{aligned} & 44.1 \\ & 14.3 \\ & 17.7 \end{aligned}$ | 3,251,085. 35 <br> 657,762.78 <br> 2,614,862,86 |
| Massachusetts Michigan. Minnesota | $\begin{array}{r} 6,597.100 \\ 12,736,227 \\ 10,656.569 \end{array}$ | $\begin{array}{r} 17.500 .00 \\ 1,105.724 .69 \end{array}$ | $\begin{array}{r} 17.500 .00 \\ 1.105 .724 .69 \end{array}$ |  | $161.9$ | $\begin{aligned} & 3,075,711.12 \\ & 2,320,320.00 \\ & 3,242,997.63 \end{aligned}$ | $\begin{aligned} & 2,656,4144.21 \\ & 2,318,670.20 \\ & 3,235,250.48 \end{aligned}$ | 412,620.05 | $\begin{aligned} & 15.4 \\ & 33.8 \\ & 32.7 \end{aligned}$ | $\begin{array}{r} 52.9 \\ 117.5 \\ 469.7 \end{array}$ | $\begin{array}{r} 273,771.01 \\ 3.754,701.00 \\ 1.474,064.52 \end{array}$ | $\begin{array}{r} 6.1 \\ 189.4 \\ 199.1 \end{array}$ | $\begin{aligned} & 3.6666 .824 .78 \\ & 6.645 .356 .70 \\ & 4,541,529.31 \end{aligned}$ |
| Mississippi <br> Missouri <br> Montana | $\begin{array}{r} 6.978,675 \\ 12.180,306 \\ 7.439 .748 \end{array}$ | 16,001.35 | 16.001. 35 |  | . 5 | $\begin{aligned} & 1,162,776.36 \\ & 3,907,101.05 \\ & 3.575,421.30 \end{aligned}$ | $\begin{array}{r} 801,57.63 \\ 3,58,50.63 \\ 3.395,421.07 \end{array}$ | 322.573 .79 $100,000.00$ |  | $\begin{array}{r} 9.4 \\ 234.2 \\ 273.9 \end{array}$ | $\begin{array}{r} 312,984.42 \\ 1,979.657 .16 \\ 1.056 .141 .73 \end{array}$ | $\begin{array}{r} 30.9 \\ 173.2 \\ 102.6 \end{array}$ | $\begin{aligned} & 5,864,119.95 \\ & 6.617,60.97 \\ & 2,972,183.62 \end{aligned}$ |
| Nebraska <br> Nevada <br> New Hampshire | $\begin{aligned} & 7.828 .961 \\ & 4.545 .917 \\ & 1.909 .839 \end{aligned}$ | 143.918 .11 24.027 .31 | $\begin{array}{r} 125,832.56 \\ 24,027.31 \end{array}$ |  | $\begin{aligned} & 45.8 \\ & 10.8 \end{aligned}$ | $\begin{array}{r} 5.120,648.07 \\ 1.544,956.41 \\ 869,951.82 \\ \hline \end{array}$ | 4.334.969.61 <br> $1.524,956.41$ 869.951 .82 | 108.538.00 | $\begin{aligned} & 29.9 \\ & 19.5 \\ & 22.9 \end{aligned}$ | $\begin{array}{r} 496.4 \\ 204.9 \\ 21.5 \end{array}$ | $976,701.91$ $846,182.27$ 175.009. 86 | $\begin{array}{r} 86.0 \\ 40.3 \\ 8.3 \end{array}$ | $\begin{array}{r} 2,391.456 .92 \\ 2,150,751.0 \\ 844,877.32 \end{array}$ |
| New Jersey New Mexico New York | $\begin{array}{r} 6.346 .039 \\ 5.792 .935 \\ 22.330 .101 \\ \hline \end{array}$ | $\begin{array}{r} 164,325.27 \\ 26,300.00 \end{array}$ | $164,325.27$ 26.300 .00 |  | $\begin{array}{r} 26.7 \\ .5 \end{array}$ | $\begin{array}{r} 1.387 .179 .29 .29 \\ 2.883 .456 .85 \\ 15.423 .937 .99 \\ \hline \end{array}$ | $\begin{array}{r} 1,387.179 .29 \\ 2,893.456 .85 \\ 14.879 .563 .19 \end{array}$ | 300,000.00 | $\begin{aligned} & 25.4 .4 \\ & 17.5 \\ & 15.2 \end{aligned}$ | 13.4 44.6 286.4 | $\begin{array}{r} 1,931,310.10 \\ 439.504 .91 \\ 6.021,850.00 \end{array}$ | $\begin{array}{r} 16.0 \\ 246.9 \\ 81.7 \end{array}$ | $\begin{aligned} & 3.027,549.61 \\ & 2,295,647.97 \\ & 1,402,387.81 \end{aligned}$ |
| North Carolina North Dakota Ohio. | $\begin{array}{r} 9,522,293 \\ 5.8044448 \\ 15.454,592 \end{array}$ | $\begin{array}{r} 118,146.93 \\ 217.402 .66 \\ 4,090.00 \end{array}$ | $\begin{array}{r} 67.958 .93 \\ 217.402 .66 \\ 4.090 .00 \end{array}$ | 50,188.00 | $\begin{array}{r} 17.1 \\ 146.8 \\ 1 \end{array}$ | $\begin{aligned} & 1,935599.23 \\ & \text { 1.041.764. } 23 \\ & 6.580,290.00 \end{aligned}$ | $\begin{aligned} & 1,411,454.77 \\ & 1.041 .744 .37 \\ & 6.384,879.90 \end{aligned}$ | 518,662.69 | $\begin{aligned} & 22.3 \\ & \begin{array}{l} 26.0 \\ 27.0 \end{array} \end{aligned}$ | $\begin{aligned} & 189.5 \\ & 304.8 \\ & 316.4 \end{aligned}$ | 1,233.884. 64 <br> 1.329,013.83 <br> 4.732,155.00 | $\begin{aligned} & 158.5 \\ & 316.9 \\ & 134.9 \end{aligned}$ | $\begin{aligned} & 6,808,994.66 \\ & 3.216,267.14 \\ & 4.363 .467 .10 \end{aligned}$ |
| Oklahoma <br> Oregon <br> Pennsylvania | 9.216.798 <br> 6.106,896 <br> 18,891,004 | 137,000.56 | 123,632.06 |  | 30.9 | 2, $584,618.52$ <br> 2, $680,955.63$ <br> 6.750,623. 31 | $\begin{aligned} & 2,884,618.52 \\ & 2,650,534.63 \end{aligned}$ $6.667 .531 .22$ |  | $\begin{aligned} & \begin{array}{l} 5.8 \\ 16.1 \\ 16.2 \end{array} \end{aligned}$ | $\begin{aligned} & 170.6 \\ & 115.9 \\ & 538.7 \end{aligned}$ | 1,346,675.15 <br> 1.136,430.88 <br> 6,582,624. 74 | $\begin{array}{r} 74.5 \\ 10.7 \\ 1064.7 \\ \hline \end{array}$ | 4,985.504. 33 <br> 2,146.298.43 <br> $5.640,848.04$ |
| Rhode Island South Carolina South Dakota | $\begin{aligned} & 1.998,708 \\ & 5.459,165 \\ & 6.011,479 \end{aligned}$ | 558.953.88 | 558.953.88 |  | 63.8 | $\begin{aligned} & 947.533 .05 \\ & 1.940 .174 .17 \\ & 1.523 .708 .87 \end{aligned}$ | $\begin{array}{r} 947.533 .05 \\ 1.90 .029 .54 \\ 1.488,826.75 \end{array}$ | $\begin{aligned} & 3,134,63 \\ & 34,882,12 \end{aligned}$ | $\begin{aligned} & 23.2 \\ & 12.3 \\ & 10.8 \end{aligned}$ | $\begin{array}{r} 19.8 \\ 224.3 \\ 246.1 \end{array}$ | $\begin{aligned} & 247,517.15 \\ & 625,665.75 \\ & 479.354 \cdot 96 \end{aligned}$ | $\begin{aligned} & 4.6 \\ & 71.5 \\ & 25.0 \end{aligned}$ | $\begin{array}{r} 803,657.80 \\ 2,931.459 .71 \\ 3,484,343.41 \end{array}$ |
| Tennessce Texas Utah | $8,492,619$ $24,244,024$ $24,244,024$ $4.194,708$ | 421,042. 89 | 421.042 .89 |  | 89.4 | 1.339.874. 27 <br> 6.766,934.03 <br> 2,286,455.85 | $\begin{array}{r} 915.516 .55 \\ 5.374,106.21 \\ 2,285,049.77 \end{array}$ | 424,357.72 | $\begin{aligned} & 22.7 \\ & 12.3 \\ & 48.7 \end{aligned}$ | $\begin{gathered} 46.3 \\ 710.1 \\ 221.2 \end{gathered}$ | $\begin{aligned} & 1,225,903.87 \\ & 5.018 .433 .53 \\ & 477,953.67 \end{aligned}$ | $\begin{array}{r} 91.5 \\ 605.0 \\ 15.3 \end{array}$ | $\begin{array}{r} 6,351,198.58 \\ 13.351 .484 .26 \\ 1.010,661.67 \end{array}$ |
| Vermont Virginia Washington |  | $\begin{aligned} & 41,694.23 \\ & 37,687.80 \end{aligned}$ | $\begin{aligned} & 41.694 .23 \\ & 26,918.09 \end{aligned}$ | 10,769.71 | 5.8 2.9 | $\begin{aligned} & 1,140,388.72 \\ & 2,037,649.30 \end{aligned}$ $2,312,445.28$ | $\begin{aligned} & 1,130,292.59 \\ & 2,02,475.65 \\ & 2,309,087.86 \end{aligned}$ | $\begin{array}{r} 15.173 .65 \\ 3.356 .82 \\ \hline \end{array}$ | $\begin{gathered} 7.3 .8 \\ 20.8 \\ 23.9 \end{gathered}$ | $\begin{array}{r} 60.0 \\ 146.9 \\ 166.6 \end{array}$ | $\begin{array}{r} 260.865 .87 \\ 1.774,961.56 \\ 838,445.86 \end{array}$ | $\begin{aligned} & 14.1 \\ & 60.1 \\ & 69.2 \end{aligned}$ | $\begin{array}{r} 434,723.31 \\ 3,592,401.70 \\ 2,968,363.28 \end{array}$ |
| West Virginia <br> Wisconsin <br> Wyoming | 4, 474, 234 <br> 9, 724,881 <br> 4.501.327 | $\begin{aligned} & 319,550.73 \\ & 234,014.99 \\ & \hline \end{aligned}$ | $\begin{aligned} & 319,550.73 \\ & 222,400.00 \end{aligned}$ |  | $\begin{aligned} & 12.1 \\ & 64.0 \end{aligned}$ | $\begin{aligned} & 1,773,351.47 \\ & 4, .333,36.17 \\ & 2,211,969.03 \end{aligned}$ | $\begin{aligned} & 1,773,351,47 \\ & 4,271,21,50 \\ & 2,076,009.43 \end{aligned}$ | $\begin{aligned} & 21.500 .00 \\ & 60,000.00 \end{aligned}$ | $\begin{aligned} & 30.3 \\ & 31.2 .2 \\ & 18.4 \end{aligned}$ | $\begin{array}{r} 72.7 \\ 205.6 \\ 366.9 \end{array}$ | $428,793.25$ 713.823 .31 553.603.20 | $\begin{aligned} & 15.3 .3 \\ & 34.1 \\ & 83.1 \end{aligned}$ | $\begin{aligned} & 2,272,089.28 \\ & 4.420,287.46 \\ & 1,649.314 .37 \end{aligned}$ |
| District of Columbia Hawaii | $\begin{aligned} & 1,918,469 \\ & 1,871,062 \end{aligned}$ | 113.402 .25 | 113.402 .25 |  | . 8 | 1,003,853.38 $422,882.57$ | $1.003,753.38$ $205,708.64$ | 167,174.18 | $\begin{array}{r} 2.4 \\ 2.9 \end{array}$ | $\begin{array}{r} 5.9 \\ 10.1 \end{array}$ | $\begin{aligned} & 588,429.20 \\ & 700.841 .31 \end{aligned}$ | $\begin{array}{r} 3.4 \\ 14.2 \\ \hline \end{array}$ | $\begin{array}{r} 212,784.17 \\ 964.512 .05 \\ \hline \end{array}$ |
| totals | 394,000,000 | 4.945.297.90 | 4,783.541.10 | 60.951 .71 | 765.0 | 134.491.394.79 | 124,652,052.56 | 5,070.554. 32 | 2.7 | 8,813.3 | 76,618,989.20 | 4,747.9 | 187.945.417.14 |





[^0]:    Reports on Subgrade Soil Studies, reprinted from Public Roads, vol. 12, nos. 4, 5, 7 , and 8 .

[^1]:    Stresses in Concrete Pavements Computed by Theoretical Analysis, by H. M.

[^2]:    ${ }^{2}$ See, for example, the paper, Computation of citresses in Bridge Slabs Due to Wheel Loads, by H. M. Westergatad. Puhlic Romds, vol. 11. no. 1, March 1930, especially equations 111 . 15 , and 16.
    3 see, for evample, A. and L. Föppl, 1 rang und Zwang, vol. I, 2d edition, l!d2t, p. 174, equations $\times 5$ and s6.

[^3]:    Pocket Strain Gage Gives Stresses in Concrete Roads, A. T. Goldbeck, Engincering News-Record, Mar. 29, 1923.
    ${ }^{2}$ Cf. the following publications:

    1. Impact Tests on Concrete Pavement Slabs, by Leslie W. Teller, Public Roads, Vol. 5, no. 2, April 1924.
    static Load Tests on Pa
    2. Static Load Tests on Pavement Slabs, by J. T. Thompson, Public Roads, vol. 5, no. 9, November 1924.
    3. Stress Measurements in Concrete Pavements, by L. W. Teller, Proc. 5th Annual Meeting, Highway Research Board, 1925.
    4. The Six Wheel Truck and the Parement, by L. W. Teller, Public Roads, vol. 6, no. 8, October 1925.
    5. Static and Impact Strains in Concrete, by J. T. Thompson, Public Roads, vol. 7, no. 5, July 1926
    6. Tests of the Delaware Riser Bridge Floor Slabs, by Geo. IV. Davis, Pubic ROADS, vol. 8 , no. 8 , Octoher 1927 .
[^4]:    ${ }^{3}$ See subgrade soil studies, p. 38.

